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THE CHEMICAL EVOLUTION OF THE GALAXY

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One of the more complicated problems in astrophysics is that of the chemical evolution of the galaxy. There are many different pieces of observational information which bear on this subject, and any general theory must draw on a large number of individual theories which describe the behavior of various parts of galaxies. These include not only theories of galactic structure, but also those of stellar evolution, of nucleosynthesis, of star formation, and of the behavior of the interstellar medium. As these theories are all imperfectly developed at the present time, it might be considered very premature to attempt a grand synthesis of such theories to account for the evolution of the galaxy as a whole. However, we have found it very instructive in our own research to attempt to find a consistent set of assumptions involving these various theories which predicts a large number of observed properties of the galaxy (Truran, Hansen and Cameron 1965; Truran and Cameron 1970). It is very likely that more than one successful set of such assumptions can be found; nevertheless, it is true that the behavior of one aspect of galactic evolution can bring about agreement with the observed properties of the galaxy only provided that various other aspects of different theories are also true. Thus, it is clear that any set of consistent assumptions which is put forward to explain the properties of the galaxy must stand or fall together. Such a set of assumptions provides a set of predictions which can be investigated in further research; if any of these assumptions proves to be false, then it is very likely that other assumptions are also false. Proceeding in this manner, we believe that a useful contribution to galactic research can be made.

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1

The detailed results which we have obtained in our calculations will be presented elsewhere. In this article we present one interesting set of self-consistent assumptions which has been arrived at in the course of our investigations, and we indicate the consequences of deviations from these assumptions. Most of our assumptions deal with the consequences of stellar evolution as they affect various processes of nucleosynthesis. Our stellar evolution assumptions are summarized in figure 1, where we have indicated the compositional structure of stars of various mass at the endpoint of their evolution. We also show a successful set of calculations of the abundances of different classes of elements as a function of time in the galaxy in figure 2; these abundances are all given relative to the abundances of the appropriate class of elements in the solar system. We consider these results to represent a successful evolutionary sequence of element abundances in the galaxy because these ratios are all close to unity at a time in galactic evolution which we identify with the formation of the solar system. In the following sections, we elaborate various of the assumptions leading to this rather more successful model.

Stellar Mass Loss. The many excellent theoretical calculations of stellar evolution are rather minimally useful to us in our present study. The reason is that ordinary stellar evolution calculations do not carry forward the evolution of the star to its final configuration; one is therefore forced to guess how the subsequent evolution of stars of various masses will influence the final chemical structure.

One of the greatest uncertainties in stellar evolution theory is that concerning the magnitude and influence of mass loss. Red giant stars are observed to be losing mass at a prodigious rate, often as much as 10^{-6} solar masses per year (Deutsch 1969). Current stellar evolution calculations indicate that in clusters in which stars of about 1 solar mass are turning off from the main sequence, the later horizontal branch stars have masses in the vicinity of 0.5 solar masses (Iben and Rood 1970). Consequently, mass losses of about half the original main sequence stellar mass appear to be indicated by the current interplay of theory and observation. There is no good quantitative estimate of the total amount of mass lost from more massive stars during the course of their evolution, but such stars certainly lose a good deal of mass in the course of their red giant phases. We have therefore assumed that the loss of about half of the initial main sequence stellar mass is quite common, and this assumption is reflected in the large amount of mass loss indicated for the range of stellar mass shown in figure 1.

There is evidence that stars lose even larger amounts of mass than this, since, for stars with masses in the general vicinity of one solar mass, the pre-

main-sequence T Tauri phase is one in which extremely rapid rates of mass loss occur (Kuhi 1966). However, in the stellar birthrate functions used in this study, which have been estimated by Salpeter (1955; 1959) and Limber (1960), the distribution of stellar masses on the main sequence is given. It is therefore appropriate to ignore the mass loss which occurs prior to the main sequence, since the mass involved is promptly lost back to the interstellar medium with essentially no nuclear transformation occurring within it. There is no net effect of such mass having been incorporated, however briefly, in stars at all.

Supernova Explosions. Our assumptions about the consequences, both nuclear and stellar, of supernova explosions have been strongly influenced by recent studies of supernova hydrodynamics by Arnett (1969a; see also Hansen and Wheeler 1969) and by calculations of explosive nucleosynthesis (Arnett 1969b; Truran and Arnett 1970; Arnett, Truran and Woosley 1971). These studies deal primarily with the behavior of the central core of the star; the total mass of the star which contains this core is undetermined. From these studies it appears that in cores of somewhat lower mass ($M_c \sim$ 2–4 $M_\odot$), in which carbon- or oxygen-burning has not occurred prior to the onset of electron degeneracy in the core, the thermonuclear ignition of carbon- or oxygen-burning will propagate outwards as a nuclear detonation wave, blowing the associated star completely into pieces. In cores of somewhat more massive stars, in which carbon- and oxygen-burning commenced in a non-explosive manner, we have assumed that the collapse of the core leads to so much collapsed mass that a Schwarzschild singularity, black hole, or "collapsar" is formed. Such an implosion event may or may not be accompanied by the expulsion of outer layers as a result of an accompanying thermonuclear explosion; in figure 1 we show that no such expectation of nucleosynthesis accompanying black hole formation occurs. We have assumed, finally, that in the stellar cores of smallest mass ($M_c \sim$ 1–2 $M_\odot$) the electron degeneracy may become sufficiently great that thermonuclear ignition of carbon is postponed until the collapsing core reaches such a high density that the resulting thermonuclear explosion is unable to lift the electron degeneracy, thus failing to produce a sufficient rise in pressure to reverse the implosion of the core. For these cases we have assumed that a neutron star is formed at the center, but that the outer layers are subject to a thermonuclear explosion which expels them into space.

The mass ejection mechanism appropriate to these small mass cores which we assume to give rise to neutron star remnants is extremely uncertain. At the present time, there is grave doubt that any significant amount of mass ejection can occur by means of the neutrino-antineutrino transport process studied by Colgate and White (1966) and by Arnett (1967). Recent numerical hydro-

dynamic studies of a supernova implosion by Wilson (1970), in which the neutrino and antineutrino transport was treated somewhat more correctly, failed to give any expulsion of material.

One interesting possible mechanism which is consistent with our assumption that r-process synthesis takes place in the material ejected from such events has recently been investigated by Leblanc and Wilson (1970). They have carried out a two-dimensional hydrodynamic calculation of the collapse of a star, allowing for the presence of both rotation and magnetic fields in the interior. An initially co-rotating star was found to develop differential rotation following collapse and the rotational shear in the inner part of the disc thus formed wrapped the magnetic field lines into a very tight spiral, creating an enormous magnetic energy close to the axis of the collapsed star. The excess magnetic pressure gave rise to an expansion of the material along the axis which, due to the buoyance of the material with respect to the local neighborhood, caused the ejection of a jet of material along the axis of the configuration. Their estimate of the mass contained in this jet, $\sim 10^{-2} M_\odot$, is quite consistent with our estimate of the mass of r-process material which must be ejected in such a supernova event in order to account for the observed fraction of r-process elements in solar system material.

The consequences of these various assumptions are shown more quantitatively in figure 1. We have assigned the mass ranges in which these various effects occur mainly in order to produce at the present time in our computed galactic history a total supernova rate of about 4 per century, which is approximately the inferred supernova rate in our galaxy (Tammann 1970). We have chosen the width of the stellar mass range giving rise to neutron stars ($5 \leq M \leq 6 M_\odot$) of just that sufficient size to make about half of the supernova explosions yield neutron star remnants, with the other half leaving no stellar remnant at all. In this sense, we do not include the most massive stars which yield the black holes among the supernova explosions, but if we had, the number of events involved is too small to make any significant contribution to the total number of supernova events. The reason for the assumed approximate equality between the supernova explosions producing neutron stars and those producing no remnant is our tentative identification of the events that produce neutron stars with the Type I supernovae and the events that do not produce neutron stars with the Type II supernovae.

Formation of White Dwarfs. It may be seen in figure 1 that the lowest mass assumed to form neutron stars is about 5 solar masses. It follows that mass loss must be expected to give rise to white dwarf remnants in the stars below this range. In general we have assumed that a white dwarf star will contain helium-burning products such as carbon and oxygen, but not the helium layer

which occurs in the advanced stages of stellar evolution nor any outerlying hydrogen layers. Thus, after mass loss has reduced an initial 1 solar mass star on the main sequence to about 0.5 solar masses on the horizontal branch, it is assumed that most of the remaining mass eventually forms a white dwarf star. Approximately the same mass fractions are assumed to be left as white dwarf remnants for stars slightly more massive than 1 solar mass, but we must assure that the total mass which is left in a white dwarf star does not exceed the Chandrasekhar (1939) limit of about 1.4 solar masses; hence, the mass fraction left as a white dwarf star is assumed to decrease as shown in figure 1. There is a discontinuity in the mass fractions associated with various stellar properties at the threshold for the supernova mass range at 5 solar masses, because it is assumed that stellar implosions commence only at that point.

Nucleosynthesis. The synthesis of elements from carbon through the iron equilibrium peak has been demonstrated to take place under explosive burning conditions resulting from the thermonuclear ignition and explosion of material in stellar cores at the time of supernova explosions (Truran, Arnett and Cameron 1967; Arnett 1969b; Truran and Arnett 1970; Arnett, Truran and Woosley 1971). We have assumed that this will occur in the outer layers of the stars which give rise to neutron star remnants and throughout the entire cores of the somewhat more massive stars which leave no remnants. The mass fractions assigned to this process in figure 1 have given, throughout our investigations, a very reasonable representation of the build-up of this range of elements in the galaxy, and we have made only minor adjustments in our original assumptions in this regard to improve the fit of the carbon to iron range of elements with solar abundances.

In an advanced stage of evolution, a star will contain two thermonuclear burning shells due to hydrogen and helium reactions. The helium content of the star thus lies mainly between these two shells, at least as far as the helium produced in the star is concerned (the outer hydrogen layer will still contain the original content of helium). Stellar evolution calculations indicate that typically 5 percent of the stellar mass is contained in this helium layer in a late stage of evolution (Hayashi, Hoshi and Sugimoto 1962) and this has been assumed in the construction of figure 1.

Nitrogen is an interesting secondary element (one whose production demands the prior buildup of other heavy elements) whose distribution can be studied in our own and other galaxies. It is produced when the primary carbon and oxygen are used as catalysts in the conversion of hydrogen into helium, which occurs at the relatively high temperatures involved in hydrogen-burning shells. Thus, the helium layer in a star in an advanced stage of evolution can be expected to have the initial carbon and oxygen in the star
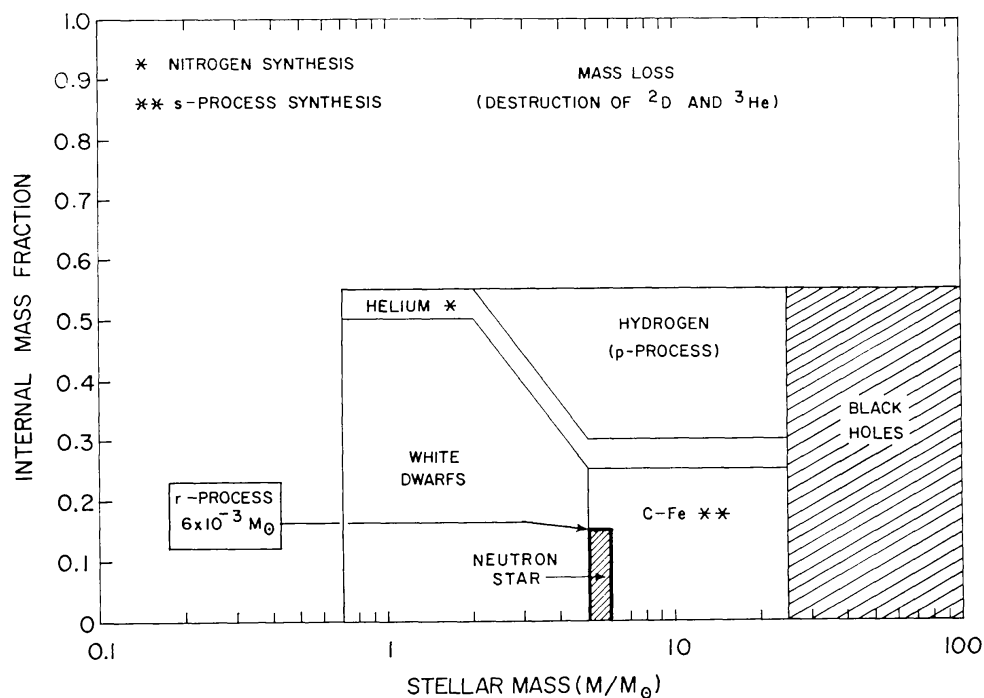
FIG. 1—The adopted compositional structures for stars of various mass in their final stages of evolution are shown. The fractional stellar mass both in the appropriate remnant and in various nuclear burning zones is indicated as a function of the main sequence mass.

converted into nitrogen. In addition, some stars in advanced stages of evolution undergo helium-burning shell flashes as a result of which there can be significant mixing of material in the helium layer into the outer layers of the star (Schwarzschild and Härm 1967), including additional nitrogen made by the carbon-nitrogen-oxygen cycles operating on the carbon produced by helium-burning and then mixed into the hydrogen-burning region. We have therefore assumed that the nitrogen produced by stellar evolution is somewhat greater than the initial carbon and oxygen content of the helium layers of the stars.

There are three processes which are primarily responsible for the manufacture of the heavy elements beyond the iron equilibrium peak. These are neutron capture on a slow time scale (*s*-process), neutron capture on a rapid time scale (*r*-process), and rapid proton capture (*p*-process). These processes occur under different circumstances; our adopted assumptions concerning them, shown in figure 1, are based upon the following considerations.

The *s*-process operates on neutron sources which are available during the ordinary and relatively slow evolution of a star. The precise neutron sources involved are still uncertain. However, there are two possible neutron sources associated with helium-burning, as well as a neutron source associated with

carbon-burning. We have assumed that the helium-burning neutron source associated with mixing together the products of helium-burning with hydrogen, which can occur as a result of the helium-burning shell flashes discussed by Schwarzschild and Harm (1967), may be the main source of *s*-process neutrons (Sanders 1967; Cameron and Fowler 1970); this also leads to the possibility that the heavy element products can subsequently be mixed to the surface of the star, where they will be lost as a part of the regular stellar mass loss process. This has the advantage that the heavy elements produced by the *s*-process are not asked to survive the extreme shock wave heating which is associated with a supernova explosion. Thus, we have assumed that the *s*-process elements are produced in stars as a small fraction of the initial amount of material in the carbon to iron element range which is exposed to this *s*-process neutron source and subsequently expelled.

The *r*-process heavy elements can be made most easily in material which has been compressed to nearly nuclear densities, such as occurs in the formation of a neutron star (Cameron, Delano and Truran 1970). The amount of material shown in figure 1 which forms *r*-process heavy elements has been entirely adjusted to fit the observed abundances of these elements in the solar system. The amount of mass, 0.006 solar masses per event from just that range of stars leading to neutron star remnants, is a very small mass fraction indeed. We are encouraged, however, by the consistency of this mass estimate with the predicted mass ejection for the supernova mechanism of Leblanc and Wilson (1970) discussed previously.

The *p*-process heavy elements are very much smaller in abundance than the heavy elements produced by either of the two neutron capture processes. It appears probable that they are produced by secondary processes, in which a supernova shock wave sweeps through the hydrogen outer layer of the pre-supernova star, raising the temperature momentarily to about 3×10^9 °K, and thus allowing protons to be rapidly added onto any heavy elements which originally were present in the star when formed and which have survived in the outer hydrogen layer. The principal difference between the outer hydrogen layer of a star which is lost by mass loss prior to a supernova explosion and that which is lost during the supernova explosion is the production of the *p*-process heavy elements in the latter.

Cosmochronology. Among the heavy elements made in nucleosynthesis are some which are radioactive with rather long half lives. These include the natural radioactivities found upon the earth, Th^{232}, U^{235}, and U^{238}. One can estimate relative production rates for these nuclides on the basis of reasonable nucleosystematics. In principle, it would be possible also to make calculations dealing with the radioactivity K^{40}, but as yet we are unable to predict with

sufficient confidence the production ratio of this nuclide relative to the other potassium isotopes; hence K^{40} does not serve as a useful indicator of cosmo-chronology at the present time.

We have also carried out the calculations keeping track of the shorter lived radioactivities I^{129} and Pu^{244}. The former has a half life of 1.7×10^7 years, and the latter has a half life of 8.2×10^7 years. Both radioactivities produce xenon isotopes, the latter as a result of a fission process. The relative amounts of these decay products can be measured in meteorites, and these relative amounts indicate the amount of I^{129} or Pu^{244} present in the meteorites at the time their parent bodies had cooled sufficiently to prevent any further diffu-sion of xenon away from the production site. It has been apparent for some time that solar system material became isolated from sources of galactic nucleosynthesis before the formation of the solar system and cooling of the meteorite parent bodies. Thus we do not expect the ratios of these shorter lived radioactivities to the amounts found in meteorites to be unity in figure 2; instead, the excess amounts of the radioactivities produced by galactic nucleosynthesis allows an estimate of the total isolation time of material which is to form the solar system before such formation actually occurred and the meteorite parent bodies cooled. The relative over-abundance factors of the two isotopes in figure 2 thus should be consistent with such an isolation time. Indeed, the over-abundances shown in figure 2 indicate an isolation time of about 8×10^7 years. This may be the last time before the cooling of the meteorite parent bodies that fresh radioactive debris from Type I supernovae, assumed to have produced these radioactivities, mixed into the interstellar cloud that would later collapse to form the sun and other stars.

In order that the ratios of U^{235} to U^{238} and Th^{232} to U^{238} should be unity at the time of formation of the solar system, we have found it necessary to assume that our galaxy is now between 1.2 and 1.3×10^{10} years old. This cosmochronological age for the galaxy is consistent with the results found by Fowler (1970).

Stellar Birth Rate Function. In a previous study of the evolution of the ele-ments in the galaxy, carried out much more crudely than the present one, Hansen, Truran, and Cameron (1965) used the birth rate function which appears to characterize the relative numbers of stars of different masses formed at the present time in the galaxy, derived from investigations by Salpeter (1959) and Limber (1960). In our present calculations we have again assumed the stellar birth rate function throughout *most* of galactic history is that observed in young clusters and in the solar neighborhood. It was also assumed, as before, that the stellar evolutionary lifetimes of the stars were essentially the lifetimes on the main sequence and were independent of the
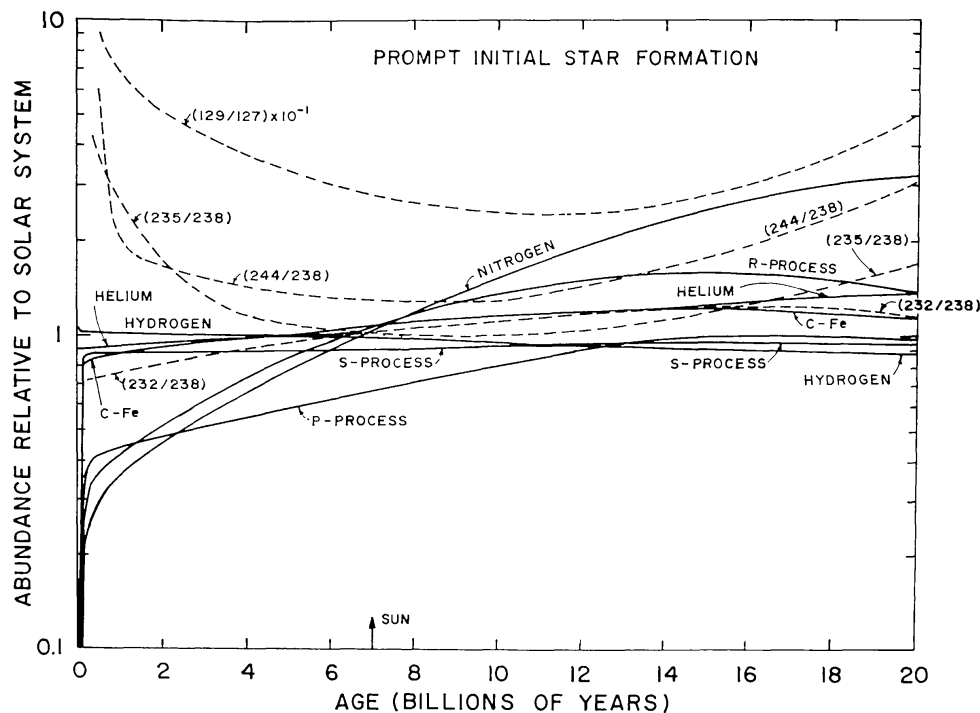
FIG. 2—The abundances of nuclei formed by the various mechanisms of nucleosynthesis, relative to their solar system values, are shown as a function of galactic age for a "successful" computed galactic history. The ratios of various radio-activities, again relative to the "primordial" solar system ratios, are also followed in time. The time of formation of the sun indicated in this figure is taken to be 5 billion years prior to the time the gas content falls to 5 percent of the galactic mass.

primordial stellar composition.

When this assumed birth rate function was used from the beginning of the galactic evolution, it was found that stars of relative low mass, which would have accumulated without having evolved off the main sequence since the beginning of galactic history, would have too wide a spread in the distributions of heavy element abundances. Thus, a typical result which we found was that some 15 percent of the stars of slightly less than solar mass would have less than 10 percent of the heavy element contents of the sun. This is quite contrary to observation, however, since the vast majority of stars in space have a heavy element content within a factor of 3 of the sun, and stars which are greatly depleted in heavy elements are actually extremely rare.

In order to reproduce this feature of galactic evolution, we have had to assume that the stellar birth rate function was drastically different in the early stages of galactic history from what it is now, as has indeed been suggested long ago by Schwarzschild and Spitzer (1953) and by Schmidt (1959, 1963). It is evident that nearly all of the stars formed with very small amounts of the heavier elements must have been sufficiently massive so that they evolved and

disappeared from the galactic scene long ago. We have therefore assumed that the birth rate function during the earliest stages of galactic evolution was the present one truncated at 10 solar masses, so that only stars of 10 solar masses and above would be formed in the galaxy until the heavy element content of the interstellar medium had built up to be one-third of that now found in the sun. This is clearly too extreme an assumption, as we do observe very old stars with metal contents of 1/10–1/1000 of the solar value—a sliding mass cutoff would have been more realistic. The resulting distribution of abundances in low mass stars is, however, in general accord with observation.

It is also reasonable on physical grounds that the birth rate function should be modified in approximately this way in the early history of our galaxy. Star formation appears to occur when an interstellar cloud is compressed to a sufficiently high density so that it goes into gravitational collapse. During the collapse, fragmentation is greatly assisted by cooling processes in the gas which depend primarily upon the presence of heavy elements. If the heavy element content is much lower, cooling will be greatly reduced, and hence fragmentation is not expected to occur as readily as it does at the present time; a preponderance of heavier mass stars in the early stages of our galactic history is therefore suggested.

Gas Content of the Galaxy. Present estimates indicate that the total gas content of our galaxy is about 5 percent of its mass (see for example, Oort 1958), even though the amount of mass in the vicinity of the sun which is in the form of gas is considerably higher than this. For want of better knowledge of the development of our galaxy, we have assumed that the total gas fraction of the galactic mass has decreased exponentially since the galaxy was formed to a present mass fraction of 5 percent. Our general procedure has been to determine by how much the total gas content of the galaxy should be decreased from one time step to the next, and then to put into stars this mass of gas together with the amount of mass evolved from stars in the same time interval. Of course, we have assumed that in the meantime there has been a thorough stirring of the mass ejected from stars with the interstellar medium.

However, this simple prescription for the history of the gas content of the galaxy has proven to be inconsistent with the detailed assumptions concerning stellar evolution and nucleosynthesis which have been outlined above. Generally speaking, we have found it impossible to produce sufficient abundances of the classes of elements made by secondary processes by the time the sun was formed to yield satisfactory agreement in figure 2; when we attempted to increase the production rates of the primary elements to improve this aspect, then we found that too many primary elements would be produced by the time of formation of the sun in our galaxy.

On the other hand, we have found that if the total amount of mass in the

galaxy at the time of formation is doubled, whereas the prescription for determining the amount of gas present thereafter is not changed, then a tremendous initial burst of star formation is imposed upon galactic history which causes the primary elements to build up extremely rapidly in the galaxy. Secondary elements can then be produced at a sufficient rate to account for solar system abundances some 7 or 8 billion years after the formation of the galaxy. This more complicated set of assumptions about the gas content of the galaxy is the basis for the successful set of abundance ratios shown in figure 2.

Discussion. This latter assumption concerning the time behavior of the gas content of the galaxy suggests some interesting physical consequences. One of the observed features of the galaxy which any galactic history must reproduce is the presence of stars in the galactic halo having a measurable, though small, content of heavy elements. If our galaxy was initially formed almost entirely out of hydrogen and helium, as most expanding cosmologies would require, then it is difficult to see how stars could have formed rapidly enough in the galactic halo to complete their stellar evolutionary lifetimes and mix their products of nucleosynthesis into the halo gases, assuming that the entire halo structure was simultaneously collapsing down toward the galactic plane. However, a tremendous early burst of star formation in the galaxy would assist in this very greatly.

According to our assumptions, such early stars would all be very massive and would evolve very rapidly. Furthermore, since the stars are made of hydrogen and helium only, they would have to obtain their energy on the main sequence primarily from the proton-proton chains, which would require that their central temperatures lie in the vicinity of 10^8 °K. These stars would therefore be more compact than usual, and their surface temperatures are likely to be in the vicinity of 10^5 °K. Such stars will radiate tremendous amounts of ultraviolet radiation, keeping the galactic gas fully ionized and probably at a kinetic temperature in the vicinity of 10^5 °K, so that the gas will remain swollen into a roughly spherical shape which fills the galactic halo. The rapid evolution of the massive stars will introduce quite large amounts of heavy elements into the halo gases, causing great enhancement of local cooling and the general collapse toward the galactic plane, although not until some star formation has taken place in the halo.

It is a consequence of this model that a significant fraction of the total galactic mass, of the order of 15 to 20 percent, will exist in the form of black holes or collapsars. The above argument would suggest that these collapsars should exist predominantly as a spherical distribution throughout the galactic halo, where they will have only a minimal effect upon the motions of the stars in the galaxy. The actual mass fraction of the galaxy which is in the form of

collapsars is highly uncertain, and it could easily be as high as 50 percent. If this situation is general, then galactic masses may have been systematically underestimated by stellar motion techniques, and the additional masses which may be present in galaxies may help to explain why virial theorem calculations tend to indicate that small clusters of galaxies generally appear to be unbound. If a large fraction of such galactic masses should be in the form of unseen collapsars, then the difficulties may be removed and small clusters of galaxies may indeed be bound.

References

Arnett, W. D. 1967, *Can. J. Phys.*, **45**, 1621.
Arnett, W. D. 1969a, *Astrophys. Space Sci.*, **5**, 180.
Arnett, W. D. 1969b, *Ap. J.*, **157**, 1369.
Arnett, W. D., Truran, J. W. and Woosley, S. E. 1971, *Nucleosynthesis in Supernova Models: II. The ^{12}C Detonation Model*, preprint.
Cameron, A. G. W., Delano, M. D. and Truran, J. W. 1970, *The Dynamics of the Rapid Neutron Capture Process*, preprint.
Cameron, A. G. W. and Fowler, W. A. 1970, *Lithium and the S-Process in Red Giant Stars*, preprint.
Chandrasekhar, S. 1939, "An Introduction to the Study of Stellar Structure," University of Chicago Press.
Colgate, S. A. and White, R. H. 1966, *Ap. J.*, **143**, 626.
Deutsch, A. J. 1969, in "Mass Loss from Stars," Hack, M. (Ed.), D. Reidel Pub. Co., Holland.
Fowler, W. A. 1970, *New Observations and Old Nucleocosmochronologies*, preprint.
Hansen, C. J. and Wheeler, J. C. 1969, *Astrophys. Space Sci.*, **3**, 464.
Hayashi, C., Hoshi, R. and Sugimoto, D. 1962, *Progr. Theor. Phys. Suppl.*, **22**, 1.
Iben, I. and Rood, R. T. 1970, *Ap. J.*, **161**, 587.
Kuhi, L. V. 1966, *Ap. J.*, **143**, 991.
Leblanc, J. M. and Wilson, J. R. 1970, *Ap. J.*, **161**, 541.
Limber, D. N. 1960, *Ap. J.*, **131**, 168.
Oort, J. H. 1958, in "Stellar Populations," O'Connell, D. J. K. (Ed.), North Holland Pub. Co., Amsterdam.
Salpeter, E. E. 1955, *Ap. J.*, **121**, 161.
Salpeter, E. E. 1959, *Ap. J.*, **129**, 608.
Sanders, R. H. 1967, *Ap. J.*, **150**, 971.
Schmidt, M. 1959, *Ap. J.*, **129**, 243.
Schmidt, M. 1963, *Ap. J.*, **137**, 758.
Schwarzschild, M. and Harm, R. 1967, *Ap. J.*, **150**, 961.
Schwarzschild, M. and Spitzer, L. 1953, *Observatory*, **73**, 77.
Tammann, G. A. 1970, *On the Frequency of Supernovae as a Function of the Integral Properties of Intermediate and late type Spiral Galaxies*, preprint.
Truran, J. W., Hansen, C. J. and Cameron, A. G. W. 1965, *Can. J. Phys.*, **43**, 1616.
Truran, J. W., Arnett, W. D. and Cameron, A. G. W. 1967, *Can. J. Phys.*, **45**, 2315.
Truran, J. W. and Cameron, A. G. W. 1970, *Nature*, **225**, 710.
Truran, J. W. and Arnett, W. D. 1970, *Ap. J.* **160**, 181.
Wilson, J. R. 1970, UCRL report 72363.